Coefficient of Thermal Expansion in Liquid Cooling Applications

Introduction

Paraffins such as those found in Opticool Fluid may be used as heat transfer mediums in electrical cooling and thermal systems, in liquid phase cooling of electronic components, in thermal energy storage and a host of similar applications [1,2,]. Although air and water have long been employed as cost-effective heat transfer agents, paraffins have the advantage of being good electrical insulators which is important in applications where removal of heat by convection – that is to have the fluid directly in contact with heated electronic components – is desired. Paraffins have better thermal insulating capability than air-cooled systems. Their other notable properties that make paraffins useful for cooling applications include thermal and chemical stability, non-toxicity, biodegradability, and low cost.

One particular property found within paraffins involves a low coefficient of thermal expansion. Coefficient of thermal expansion is defined as the change in volume of a material per one degree Celsius change in temperature. This is an intensive property and is unique for each substance.

For applications requiring fluid to be contained in pipes or containers, this property is important to consider in choosing the fluid suited for each application. As the fluid is exposed to varying temperatures, its volume also changes and the container or pipes should be designed to accommodate this volume changes to ensure proper functioning. This helps to avoid build-up of hydraulic pressure and spills during equipment servicing as well. Because the coefficient of thermal expansion for paraffins are much lower than that of air, their use as coolants offer many advantages to modern systems designers in the unending pursuit of technological miniaturization.

The coefficient of thermal expansion of a particular substance is precisely determined via experimental methods such as dilatometric and pycnometric procedures. The first method measures the changes in volume of the object using a length sensitive device and requires that the objects are highly elastic, mostly solids or solid-like material such as pastes. The second method relies on the fact that changes in volume can be indirectly obtained from the densities of materials as temperatures change [3]. Both methodologies are extremely laborious to perform [3], however, if the fluid is well-researched and their densities are known for a certain temperature of interest, the coefficient of thermal expansion of the fluid can be estimated. Thus, volume changes can be calculated as well in the following manner.

To obtain the equation for the Coefficient of Thermal Expansion, we begin with the Ideal Gas Law:

Equation	Description
$PV = nRT$	Starting Equation
$V = \frac{nRT}{P}$	Solving for the Value of V
$dV = \frac{\partial V}{\partial T}dT + \frac{\partial V}{\partial P}dP + \frac{\partial V}{\partial n}dn$	Obtaining the derivative of V: n, P, T are for partial derivatives.
$\frac{dV}{V} = \frac{1}{V}(\frac{\partial V}{\partial T})dT + \frac{1}{V}(\frac{\partial V}{\partial P})dP + \frac{1}{V}(\frac{\partial V}{\partial n})dn$	Dividing by V; this is to obtain the change in V with respect to itself.

From the above equation, we have the Volumetric Coefficient of Thermal expansion, $\boldsymbol{\beta}$, as:
$\boldsymbol{\beta} = \frac{\mathbf{1}}{\boldsymbol{V}}(\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{T}})$**.** This is the coefficient of Thermal expansion, changing with Temperature.
For a finite change of Temperature, we can get the approximate value of the Thermal expansion coefficient as:

Equation	Description
$\boldsymbol{\beta} = \frac{1}{V}(\frac{\partial V}{\partial T})$	Starting Equation
$\boldsymbol{\beta} \approx \frac{1}{V}(\frac{\Delta V}{\Delta T})$	Partial Derivative changed to delta for finite change of temperature.
$\boldsymbol{\beta} \approx \frac{1}{V}(\frac{(V2 - V1)}{(T2 - T1)})$	Change $\Delta V = V2 - V1$ and $\Delta T = T2 - T1$
$\boldsymbol{\beta} \approx \frac{1}{m/\rho}(\frac{(m/\rho 2 - m/\rho 1)}{(T2 - T1)})$	Note that $\rho = m/V$ so it follows that: $V = m/\rho$ .. And then we substitute the value.
$\boldsymbol{\beta} \approx \frac{\rho}{m}(m\frac{(1/\rho 2 - 1/\rho 1)}{(T2 - T1)})$	Simplifying the equation (for the 1/V) and factoring out m for (V2-V1)
$\boldsymbol{\beta} \approx \rho(\frac{(1/\rho 2 - 1/\rho 1)}{(T2 - T1)})$	Cancelling out m.
$\boldsymbol{\beta} \approx \rho(\frac{(\frac{\rho 1 - \rho 2}{\rho 1 \rho 2})}{(T2 - T1)})$	Simplifying
$\boldsymbol{\beta} \approx (\frac{\rho}{\rho 1 \rho 2})(\frac{(\rho 1 - \rho 2)}{(T2 - T1)})$	Simplifying
$\boldsymbol{\beta} \approx (\frac{1}{\rho 2})(\frac{(\rho 1 - \rho 2)}{(T2 - T1)})$	Since $\rho = \rho\ 1$. **[EQTN 1]**
$\boldsymbol{\beta} \approx (\frac{1}{S2})(\frac{(S1 - S2)}{(T2 - T1)})$	Using specific gravity to simplify the equation, and knowing that specific gravity is: $s = \frac{\rho(substance)}{\rho(water)}$ **[EQTN2]**

It should be noted here that for Fluids, $\boldsymbol{\beta}$ or Coefficient of Volumetric Thermal expansion is used; while for Solids $\boldsymbol{\alpha}$ or Coefficient of Linear Thermal expansion is commonly used. With that, we will have the following formula for the fluid as:

Equation	Description
$\Delta V = V_i \beta \Delta T$	ΔV -> Delta V, difference between V_f and V_i V_f -> Final Volume V_i -> Initial Volume ΔT -> Delta T, difference between T_f and T_i T_f -> Final Temperature T_i -> Initial Temperature **[EQTN3]**
$V_f - V_i = V_i \beta (T_f - T_i)$	Expanding ΔV and ΔT
$V_f = V_i + V_i \beta (T_f - T_i)$	Calculating for the Value of V_f
$V_f = V_i[1 + \beta\left(T_f - T_i\right)]$	Simplifying **[EQTN4]**

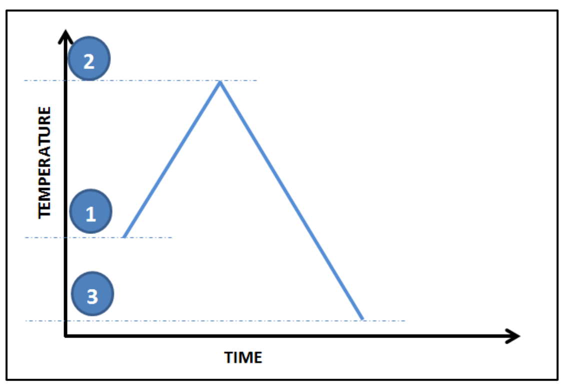
And for a solid as:

Equation	Description
$\Delta L = L_i \alpha \Delta T$	ΔL -> Delta L, difference between L_f and L_i L_f -> Final Length L_i -> Initial Length ΔT -> Delta T, difference between T_f and T_i T_f -> Final Temperature T_i -> Initial Temperature **[EQTN5]**
$L_f - L_i = L_i \alpha (T_f - T_i)$	Expanding ΔL and ΔT
$L_f = L_i + L_i \alpha (T_f - T_i)$	Calculating for the Value of L_f
$L_f = L_i[1 + \alpha\left(T_f - T_i\right)]$	Simplifying **[EQTN6]**

For an application that uses a Solid as a container for the Fluid, we will be dealing with Volume of the Solid. Thus, obtaining the relationship of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ as applied to solid will be useful, and the approximate relationship will be as follows:

Equation	Description
$V = (L\,)^3$	We will start with a cube, but this will apply to all isotropic materials, which is true for most metals. **[EQTN7]**
$V + \Delta V = (L + \Delta L\,)^3$	This will be the Volume and Length Relationship as Temperature changes, with: ΔL -> Delta L, difference between L_f and L_i L_f -> Final Length L_i -> Initial Length ΔT -> Delta T, difference between T_f and T_i T_f -> Final Temperature T_i -> Initial Temperature
$V + \Delta V =$ $L^3 + 3L^2\Delta L + 3L(\Delta L)^2 + (\Delta L)^3$	After expanding the right side of the equation.
$V + \Delta V \approx L^3 + 3L^2\Delta L$	Here we will note that this is the approximation since ΔL has a component α which is less than 1. Thus the square and the cube will approach 0 as α approaches 0 (ie ΔL^X-> 0 as α -> 0, where X is the power and X>1).
$V + \Delta V \approx V + 3V\dfrac{\Delta L}{L}$	Substituting $V = L^3$ [EQTN7] to the equation, and also $L^2 = \frac{V}{L}$
$V + \Delta V \approx V + 3V\alpha\Delta T$	Substituting [EQTN5] to the right side of the equation.
$\Delta V \approx 3V\alpha\Delta T$	Cancelling out V in both sides of the equations
$V\beta\Delta T \approx 3V\alpha\Delta T$	Substituting [EQTN3] to the left side of the equation
$\boldsymbol{\beta \approx 3\alpha}$	This equation is a good approximation for isotropic materials. **[EQTN8]**

With the relevant equations outlined, we can now apply these to a practical application. We will start by calculating the required size of a container needed to accommodate a volume of a particular liquid (Paraffin) over a known temperature range. We will use the graph shown here to indicate the temperature range of operation (given that the density at said points are known):



Equation	Description
$V_{F2} = V_{F1}[1 + \beta(T_2 - T_1)]$	Starting Equation. **[EQTN4**] Here we will calculate the Volume of the fluid when the temperature,T_2, goes up ($T_2 > T_1$) where the density, $\rho 2$, is known. We place additional notation F in the Volume variable V_{F2} , to indicate that this is for the Fluid.
$V_{C2} = V_{F2} = V_{F1}[1 + \beta(T_2 - T_1)]$	For this application, to get the required container size, it should accommodate the expanded Volume of the Fluid V_{F1}. To do that, we should have: $V_{C2} = V_{F2}$ This explains the equation on the left.
$V_{C2} = V_{F1}[1 + (\frac{1}{\rho 2})(\frac{(\rho 1 - \rho 2)}{(T2 - T1)})(T_2 - T_1)]$	Substituting value of β using [EQTN1] and simplifying the equation. NOTE: - We are using β since we are dealing with Fluid - We used [EQTN1] since datasheets usually gives the ρ of the fluid at t given Temperature T.
$V_{C2} = V_{F1}[1 + (\frac{(\rho 1 - \rho 2)}{\rho 2})]$	Simplifying
$V_{C2} = V_{F1}[\frac{\rho 1}{\rho 2}]$	Further simplification. This means that the Container Volume will be a function of the Fluid's Initial and Final Volume (with reference to the temperature graph above) over the temperature range which it will be subjected. **[EQTN9]**

For further analysis, we will calculate the Void space (volume) of the Container when the liquid completely cools down. This is the state #3 in the above graph. It should be noted that the locations of the said temperature points are arbitrary as long as density at that point is known. For this analysis, we assume that T1 = T3 (that applies to the full temperature range of the Fluid).

Equation	Description
$V_{VOID} = V_{C3} - V_{F3}$ $V_{VOID} = V_{C3} - V_{F1}$	Initial Equation. Here we know that the Volume (void) is just the difference of the Container Volume (cooled down) and Fluid Volume (cooled down) Because we know that T1 = T3 then: $V_{F3} = V_{F1}$.
$V_{VOID} = \{V_{C2}[1 + \beta_C(T_3 - T_2)]\} - V_{F1}$	Substituting [EQTN4] for the Volume of the Container. We will note that here we used the Volumetric Coefficient of Thermal expansion β_C, thus the subscript C.
$V_{VOID} = \{V_{F1}[\frac{\rho 1}{\rho 2}][1 + 3\alpha(T_1 - T_2)]\} - V_{F1}$	Substituting the Value of V_{C2} from [EQTN9]. Using [EQTN8] so that Linear Coefficient of thermal expansion, which is usually known for solid, can be used. Substituting T1 to T3 since T3=T1 here.
$V_{VOID} = V_{F1}\{[\frac{\rho 1}{\rho 2}][1 + 3\alpha(T_1 - T_2)] - 1\}$	After simplification. This gives the generic equation to get the Volume Void when the Fluid Cools down. This will also give the user an idea of the maximum Volume that can fill the Container. **[EQTN10]**

Specific Example

For the case of Paraffin (C_{16}-C_{28}), we have the Following:

Required Container Size	
Equation	**Description**
$V_{C2} = V_{F1}[\frac{\rho 1}{\rho 2}]$	Initial Equation using [EQTN9]
$V_{C2} = [\frac{0.91}{0.765}]V_{F1}$	Substituting the known values from Table 4 PCM Document. $\rho 1 = 0.910 kg/m3$ $T1 = 20\ degC$ $\rho 2 = 0.765 kg/m3$ $T2 = 70\ degC$
$V_{C2} = 1.1895 V_{F1}$	After simplification. This means that above the stated temperature range, the Final Container Volume will be around 19% more with respect to the Fluid's Initial Volume.

Void Volume	
Equation	**Description**
$V_{VOID} = V_{F1}\{[\frac{\rho 1}{\rho 2}][1 + 3\alpha(T_1 - T_2)] - 1\}$	Starting Equation [EQTN10]
$V_{VOID} = V_{F1}\{[\frac{0.91}{0.765}][1 + 3(0.000013)(20 - 70)] - 1\}$	Substituting the known values: $\rho 1 = 0.910 kg/m3$ $T1 = 20\ degC$ or 293.15 degK $\rho 2 = 0.765 kg/m3$ $T2 = 70\ degC\ or\ 323.15\ degK$ Here we will assume we have Steel as the Container's material, thus: $\alpha = 0.000013 \frac{m}{m} K$
$V_{VOID} = 0.1872 V_{F1}$	Approximately 18.72% of the Initial Fluid Volume will be the Void upon Cool down.

Conclusion:

Mineral oils are a mixture of hydrocarbons constituting saturated straight chain and cyclic hydrocarbons as well as aromatic hydrocarbons. Paraffins are purified portions of mineral oils and consist primarily of “paraffinic” or saturated straight chain hydrocarbons. Paraffins are also further classified to many fractions having specific flash/fire points, depending upon the number of carbons in the backbone of the constituent hydrocarbon for a certain fraction. In real-world cooling applications, densities of a specific paraffin fraction at a certain temperature should be accurately identified in order to accurately estimate the coefficient of thermal expansion and volume changes using the equations above. While some values may be obtained via textbooks and information found on the Internet, more reliable data may be obtained from suppliers or manufacturers for each fluid under evaluation.

Opticool Fluid

OptiCool Fluid is an isoparaffin-based dielectric heat transfer fluid manufactured by DSI Ventures, Inc. OptiCool has been used for over 10-years to cool electrical circuitry in transformers, RF and microwave transmission devices and computer systems.

OptiCool Fluid is a colorless, odorless, food grade isoparaffin oil. With a very low viscosity and high thermal conductivity, OptiCool Fluid has extremely high heat transfer coefficients, making it ideal for removing heat from circuitry with high heat flux densities.

Contact DSI to find out more about OptiCool Fluid and its electronics cooling applications.

sales@dsiventures.com
Tel: 903-526-7577

Reference:

1. N. Ukrainczyk, S. Kurajica, and J. Sipusie. "Thermophysical Comparison of Five Commercial Paraffin WSaes as Latent Heat Storage Materials." Chem. Biochem. Eng. . 24 (2) 129-1377 (2010).

2. B. Zalba, J. Marin, L. Cabea and H. Mehling. "Review on thermal energy storage with phase change: materials, heat transfer analysis and application."Applied Thermal Engineering 23 (2003) 251-283.

3. M. Schimmelpfenning, K. Weber, F. Kalb, K.-H. Feller, T. Butz and M. Matthai. "Volume expansions of paraffins from dip tube measurements."